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13. ABSTRACT (Maximum 200 words)

During this period substantial progress has been made in a number of areas, including the question of the reason for the existence of period doubling cascades; other areas include predictability and fractal basin boundaries; a new scaling effect concerning the way strange attractors are destroyed; and work in which quasiperiodic motions are to be expected in nature. This report summarizes the progress made and lists reports and papers resulting from the research performed during this period.

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Interim report on grant AFOSR-81-0217
for the period ending 30 September 1983.

I am pleased to report that substantial progress has been made

in a number of areas, including the question of the reason for the

existence of period doubling cascades. A long paper on this topic is

almost complete. A short paper giving sketches of proofs has been

accepted for publication.

Title: Cascades of Period-Doubling Bifurcations: A Prerequisite
         for Horseshoes

Authors: J. A. Yorke and K. T. Alligood

Description: Since Smale (6) described the complicated dynamical
              behavior of horseshoe maps, many dynamical systems
              depending on a parameter have been shown to develop
              horseshoes. In the horseshoe, there are 2^k fixed
              points for the kth iterate of the map. Numerical
              studies of parameterized maps and the investigations
              of Newhouse (5) indicate there is a rich structure
              of attractors for parameters preceding the existence
              of the horseshoe. Once the horseshoe if formed, however,
              all periodic points are unstable and almost any
              trajectory starting in the horseshoe eventually leaves
              it. Assuming that cross-sectional areas are con-
              tracting, we prove that infinitely many cascades of
              period doublings must occur in the process of forming
              a horseshoe. Each such cascade need not evolve
              regularly or monotonically, but it must contain
              attracting periodic points of all the periods k,
              2k, 4k, 8k, ..., for some k.

During the period under review we started and completed a paper

which connects predictability and fractal basin boundaries.

Title: Final State Sensitivity: An Obstruction to Predictability

Authors: C. Grebogi, S. W. McDonald, E. Ott, J. Yorke

Description: In situations where nonlinear dynamical systems
              possess more than one time-asymptotic final state,
              very accurate specification of the initial conditions
              may sometimes be necessary for the reliable prediction
              of the final state that is eventually reached. A
              notion of final state sensitivity is introduced in
              terms of the scaling of the final-state-uncertain

phase space volume with uncertainty in initial
conditions. The relation of final state sensitivity
to the existence of basin boundaries with noninteger
(fractal) dimension is discussed, and the implications
for prediction are illustrated by a numerical experi-
ment and by suitable interpretation of previous
theoretical and experimental work.

A paper has been completed describing a new scaling effect concerning

the way strange attractors are destroyed.

Title: Truncated Development of Chaotic Attractors when The
Jacobian is not Small

Authors: T. Short and J. Yorke

Reference: This is to be included in the Springer Verlag
(or is it Academic Press) Proceedings of the
Kyoto Japan conference "Chaos and Statistical
Mechanics"

We also completed work on which quasiperiodic motions are to be

expected in nature.

Title: Are Three-Frequency Quasiperiodic Orbits to Be Expected
in Typical Nonlinear Dynamical Systems?

Authors: C. Grebogi, E. Ott, J. Yorke

Description: The current state of theoretical understanding related
to the question posed in the title is incomplete.
This paper presents results of numerical experiments
which are consistent with a positive answer. These
results also bear on the problem of characterizing
possible routes to chaos in nonlinear dynamical systems.

Reference: Physical Review Letters, 1 August 1983, Vol. 51 #5,
pp. 339-342.

The paper on Nowhere Differentiable Attracting Tori has been

greatly extended (Revised August 1983) and gives much stronger evidence

in favor of the Dimension Conjecture of Kaplan and Yorke.

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Title: The Lyapunov Dimension of a Nowhere Differentiable
Attracting Torus

Authors: J. L. Kaplan, J. Mallet-Paret, J. A. Yorke

Reference: Accepted for publication in Ergodic Theory and
Dynamical Systems.

Title: Fractal Basin Boundaries, Long-Lived Chaotic Transients,
and Unstable-Unstable Pair Bifurcation

Authors: C. Grebogi, E. Ott, J. Yorke

Reference: Physical Review Letters, 28 March 1983, Vol. 50, pp. 935-939.

Description: A new type of bifurcation to chaos is pointed out and
discussed. In this bifurcation two unstable fixed
points or periodic orbits are created simultaneously
with a strange attractor which has a fractal basin
boundary. Chaotic transients associated with the
coalescence of the unstable-unstable pair are shown
to be extraordinarily long-lived.

In this paper we consider a new type of bifurcation
to chaotic motion. This bifurcation is characterized
by the simultaneous appearance of a pair of unstable
fixed points or periodic orbits, a fractal (i.e.,
nondifferentiable) basin boundary, and a chaotic
attractor (also commonly called a strange attractor).
Just prior to this type of bifurcation, transient
behavior with a chaotic character can take place, and
these chaotic transients can be extremely long. The
existence of such remarkably long-lived chaotic transients
may have important implications for experiments on
chaotic systems.

The papers with Alligood and the three with Grebogi are short papers
for which much longer versions are in preparation.